BRIEF COMMUNICATION

BOUNDARY LAYER FLOW OF A PARTICULATE SUSPENSION PAST A FLAT PLATE

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INTRODUCTION

This paper is concerned with steady laminar boundary layer flow of a particle/fluid suspension past a semi-infinite flat plate. The fluid is assumed to be incompressible and the volume fraction of particulate material is assumed to be small. Osiptsov (1980) and Prabha & Jain (1980) have reported numerical solutions to this problem based on the dusty gas model (e.g. Marble 1970). [Wang & Glass (1988) recently reported numerical solutions for a compressible fluid phase.] While the dusty gas model is widely used, it is by no means the only plausible representation of small volume fraction suspension behavior. It is, therefore, of interest to determine how solutions based on more elaborate models compare with those associated with the dusty gas model. In this way information can be obtained which is helpful in matching models with observed physical behavior. In the present work the title problem is solved employing a model which incorporates ideas similar to those discussed by Soo (1967, 1968) and Korjack and various coworkers (e.g. Korjack & Chen 1980). This model endows the particulate phase with diffusivity and viscosity (not present in the dusty gas model), while retaining the small volume fraction assumption.

GOVERNING EQUATIONS

Let the plate occupy the half of the x, z plane corresponding to $x > 0$ with the y axis being normal to the plate. Let u_c, v_c, ρ_c and v_c denote the fluid phase tangential velocity, normal velocity, density and kinematic viscosity, respectively. Let u_d, v_d, ρ_d, v_d and D_d denote the particle phase tangential velocity, normal velocity, density, kinematic viscosity and diffusivity, respectively. Let the free stream conditions be denoted by u_∞ and $\rho_{d\infty}$ and the momentum relaxation time by τ. Then, the equations which form the basis for the present work can be written as

$$\partial_x u_c + \partial_y v_c = 0, \tag{1a}$$

$$u_c \partial_x u_c + v_c \partial_y u_c = v_c \partial_{yy} u_c + (\rho_d/\rho_c)(u_d - u_c)/\tau, \tag{1b}$$

$$\partial_x(\rho_d u_d) + \partial_y(\rho_d v_d) = D_d \partial_{yy} \rho_d, \tag{1c}$$

$$u_d \partial_x u_d + v_d \partial_y u_d = v_d[\partial_{yy} u_d + \partial_y(\ln \rho_d)\partial_y u_d] + (u_c - u_d)/\tau \tag{1d}$$

and

$$u_d \partial_x v_d + v_d \partial_y v_d = v_d\{2[\partial_{yy} v_d + \partial_y(\ln \rho_d)\partial_y v_d] + \partial_{xy} u_d + \partial_x(\ln \rho_d)\partial_y u_d\} + (v_c - v_d)/\tau. \tag{1e}$$

In writing the above equations the usual boundary layer approximations were employed, external body forces were neglected, and, for simplicity, the quantities ρ_c, v_c, v_d, D_d and τ were treated as constants. The equations employed by Osiptsov (1980) and Prabha & Jain (1980) are recovered by equating D_d to zero in [1c] and v_d to zero in [1d, e]. One (of several) justifications for the inclusion of the terms multiplied by these two parameters is that they can be thought of as logical consequences of the averaging processes used to develop a continuum model of a system containing

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discrete elements. Substituting the modified Blasius variables

$$x = u_\infty \tau \xi/(1 - \xi), \quad y = [2v_c \tau \xi/(1 - \xi)]^{1/2}\eta,$$

$$u_c = u_\infty F_c(\xi, \eta), \quad v_c = [v_c(1 - \xi)/(2\tau\xi)]^{1/2}[G_c(\xi, \eta) + \eta F_c(\xi, \eta)],$$

$$u_d = u_\infty F_d(\xi, \eta), \quad v_d = [v_c(1 - \xi)/(2\tau\xi)]^{1/2}[G_d(\xi, \eta) + \eta F_d(\xi, \eta)],$$

$$\rho_d = \rho_{d\infty} Q_d(\xi, \eta) \tag{2}$$

into [1a–e] yields:

$$\partial_\eta G_c + F_c + 2\xi(1 - \xi)\partial_\xi F_c = 0, \tag{3a}$$

$$G_c\partial_\eta F_c + 2\xi(1 - \xi)F_c\partial_\xi F_c = \partial_{\eta\eta} F_c + 2\kappa\xi Q_d(F_d - F_c)/(1 - \xi), \tag{3b}$$

$$G_d\partial_\eta Q_d + 2\xi(1 - \xi)F_d\partial_\xi Q_d + [\partial_\eta G_d + F_d + 2\xi(1 - \xi)\partial_\xi F_d]Q_d = \delta\partial_{\eta\eta} Q_d, \tag{3c}$$

$$G_d\partial_\eta F_d + 2\xi(1 - \xi)F_d\partial_\xi F_d = \beta[\partial_{\eta\eta} F_d + \partial_\eta(\ln Q_d)\partial_\eta F_d] + 2\xi(F_c - F_d)/(1 - \xi), \tag{3d}$$

and

$$G_d\partial_\eta G_d + 2\xi(1 - \xi)F_d\partial_\xi G_d - \eta F_d^2 = \beta\{2[\partial_{\eta\eta} G_d + \partial_\eta(\ln Q_d)\partial_\eta G_d] + 3\partial_\eta F_d$$
$$+ \partial_\eta(\ln Q_d)F_d + 2\xi(1 - \xi)[\partial_{\xi\eta} F_d + \partial_\xi(\ln Q_d)\partial_\eta F_d]\} + 2\xi(G_c - G_d)/(1 - \xi). \tag{3e}$$

In [3a–e],

$$\beta = v_d/v_c, \quad \delta = D_d/v_c, \quad \kappa = \rho_{d\infty}/\rho_c \tag{4}$$

are the viscosity ratio, the inverse Schmidt number and the free stream particle loading, respectively. The boundary conditions employed were:

$$F_c(\xi, \eta) \rightarrow 1, \quad F_d(\xi, \eta) \rightarrow 1, \quad Q_d(\xi, \eta) \rightarrow 1 \quad \text{as} \quad \eta \rightarrow \infty, \tag{5a}$$

$$G_d(\xi, \eta) \rightarrow G_c(\xi, \eta) \quad \text{as} \quad \eta \rightarrow \infty, \tag{5b}$$

$$F_c(\xi, 0) = 0, \quad G_c(\xi, 0) = 0 \tag{5c}$$

and

$$F_d(\xi, 0) = \omega[(1 - \xi)/(2\xi)]^{1/2}\partial_\eta F_d(\xi, 0), \quad G_d(\xi, 0) = 0, \partial_\eta Q_d(\xi, 0) = 0, \tag{5d}$$

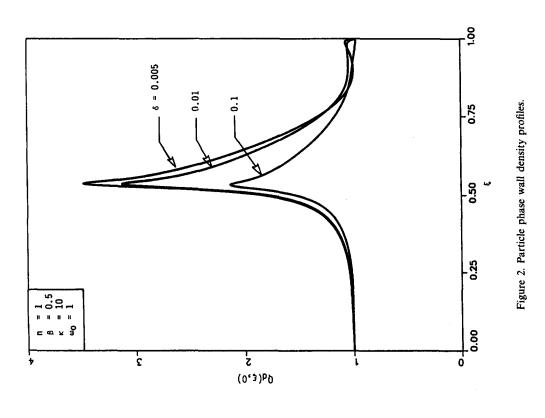where ω is a particle phase slip parameter.

In reality the particle phase tangential velocity at the wall is controlled by a variety of physical effects such as sliding friction, the nature of particle/surface collisions etc. It is not possible to model such effects with precision at present. It was, however, desired to provide enough generality in the equations to allow for a wide variety of particle phase wall tangential velocity profiles. This was accomplished by allowing the slip coefficient ω to depend on the particulate wall relative velocity $F_d(\xi, 0)$. Tentatively a function of the form
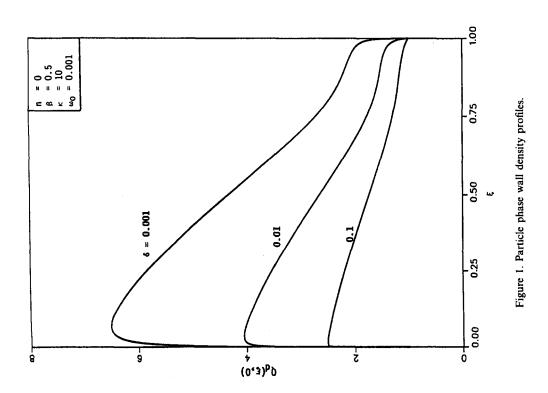
$$\omega = \omega_0\{F_d(\xi, 0)/[1 - F_d(\xi, 0)]\}^n \tag{6}$$

was employed in which ω_0 and n were treated as constants. According to [6], $\omega = \infty$ at $\xi = 0$ $[F_d(0, 0) = 1]$, where perfect slip should exist, and $\omega = 0$ at $\xi = 1$ $[F_d(1, 0) = 0]$, where no slip should exist.

RESULTS AND DISCUSSION

Equations [3a–e] were solved, subject to [5a–d], by a standard implicit finite difference method for boundary layer equations modified for application to two-phase flows. Some typical particle phase density profiles are shown in figures 1 and 2. Both Osiptsov (1980) and Prabha & Jain (1980) report (based on the dusty gas model) that the particle phase density becomes infinite at $\xi = 0.5$; i.e. at one relaxation length from the leading edge. This singularity is associated with the existence of a particle phase stagnation point at $\xi = 0.5$. In contrast to this, it can be seen that the present model predicts a singularity-free solution throughout the flowfield as long as diffusivity is included

Figure 2. Particle phase wall density profiles.

Figure 1. Particle phase wall density profiles.

in the model. As the inverse Schmidt number δ is decreased larger and larger peaks are observed in the particle phase density profiles. The locations of these peaks depend on the slip parameters ω_0 and n (because these influence the particle phase wall slip distribution). Although the corresponding results are not presented graphically, it is of interest that other parameters (skin friction, displacement thickness etc.) were affected hardly at all by the changes in δ indicated in figures 1 and 2.

CONCLUSION

It was shown that solutions for steady laminar boundary layer flow of a particle/fluid suspension past a flat plate based on a model including particle phase viscous and diffusive effects do not exhibit the singular behavior associated with the dusty gas model. Because of this singular behavior, dusty gas predictions are not self-consistent (the small volume fraction assumption being violated). The self-consistent nature of the predictions of the present model suggests that models of this general type should receive more attention in the future.

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